Analytical solutions for the coupled heat and mass transfer in flat and hollow fiber membrane-based adiabatic sorber beds

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Abstract
Adiabatic membrane-based sorber beds offer higher absorption rates than conventional sorber beds for absorption heat pump/chiller application. In this study, we propose two analytical solutions for the coupled heat and mass transfer in flat and hollow fiber membrane-based adiabatic sorber beds. The similarity solution and the Laplace transform method are used to develop the present analytical models. The proposed analytical models are validated with experimental data and numerical results available in the literature with an average relative difference of 12%.

Keywords: absorber beds; sorption reactors; absorption chillers and heat pumps; analytical solution; heat and mass transfer; hollow fiber membrane; and membrane technology.

1. Introduction

Recently, membrane-based sorber beds have received immense attention, as they can improve the coefficient of performance (COP) and reduce the size of absorption chillers/heat pumps. Membrane-based sorber beds can be categorized into two types: i) isothermal membrane-based sorber beds in which the solution film is continuously cooled or heated via a heat transfer fluid [1]; and ii) adiabatic membrane-based sorber beds in which the solution film is not cooled or heated via a heat transfer fluid during absorption process [2]. Isothermal membrane-based sorber beds have shown promising performance. While the adiabatic membrane-based sorber beds have exhibited comparable performance with their isoflux counterparts, they are more compact and less costly. Therefore, we focus on adiabatic membrane-based absorber beds.

In this paper, we developed two new analytical closed-form solutions for heat and mass transfer in flat and hollow fiber membrane-based adiabatic sorber beds used in absorption chillers and heat pumps. To develop these models, two approaches are used: i) the similarity solution; and ii) the Laplace transform method. The models are validated with the numerical studies and experimental data available in the literature.

2. Problem description

Coupled heat and mass transfer in flat and hollow fiber membrane-based adiabatic sorber beds are investigated. LiBr-water is selected as the working fluid, which is the most common solution in absorption chillers/heat pumps; however, the results can be used for other refrigerants. As schematically showed in Fig. 1, in a flat membrane-based adiabatic sorber bed, the LiBr-water solution film is confined by a plate and a microporous/nanofiber membrane. However, in a hollow fiber membrane-based adiabatic sorber bed, the LiBr-water solution flows in hollow fiber membranes. The membrane is impermeable to the LiBr-water solution but allows water vapor to pass, resulting in vapor absorption or desorption at the membrane-solution interface.

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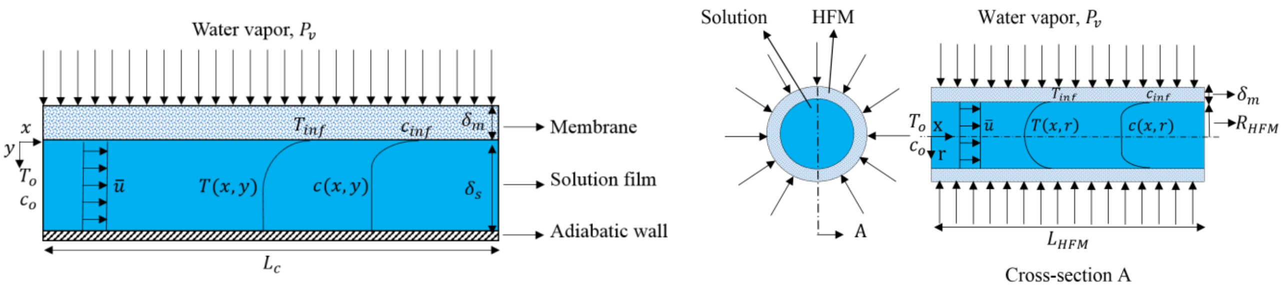


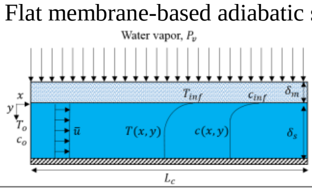
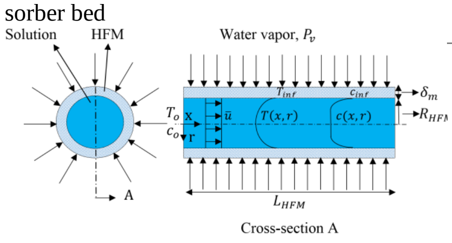
(a) (b)

Fig. 1. A schematic diagram of: (a) a flat membrane-based adiabatic sorber bed; and b) a hollow fiber membrane-based (HFM) adiabatic sorber bed.

3. Analytical solutions

To develop the analytical models, the following assumptions are made: The solution film is laminar; the flow is hydrodynamically fully developed; linear estimation is used to find the pressure at the solution-membrane interface; the mean velocity is utilized; the thermo-physical properties of LiBr-water solution are constant; heat transfer from the film to the gaseous phase and membrane is negligible; inlet concentration and temperature distributions are uniform and constant; the absorbent is non-volatile; membrane temperature is constant; and desorption temperature should be less the boiling point of the solution (i.e. nearly 95˚C for aqueous LiBr). The developed closed-form solutions are shown in Table 1.

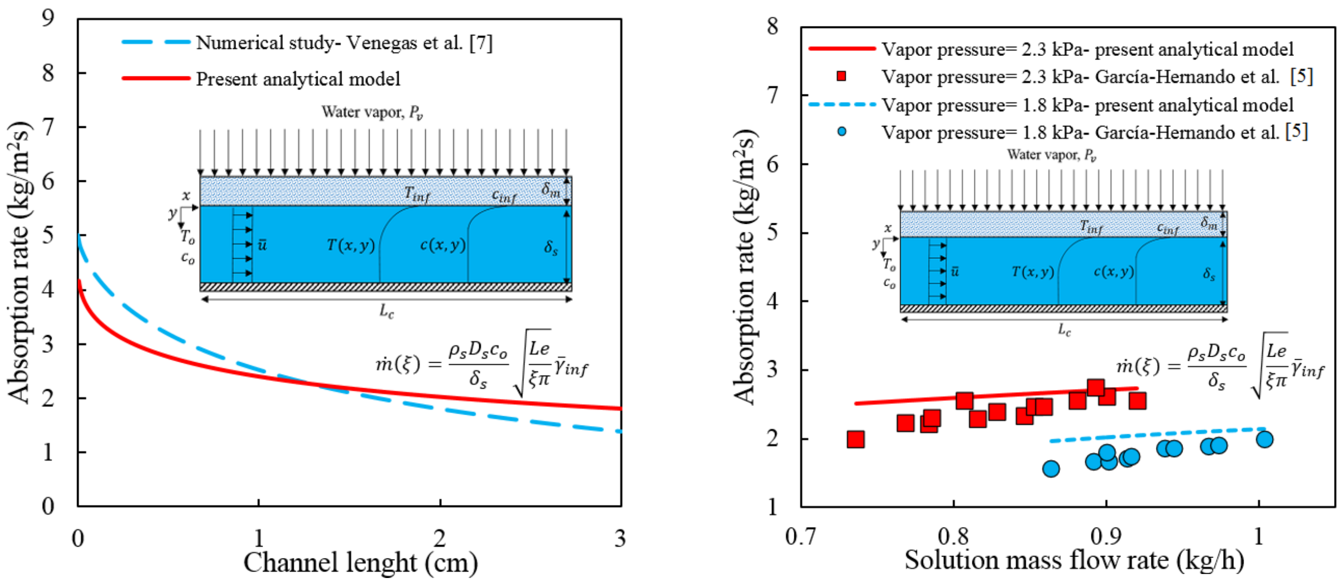
Table 1. The proposed analytical solutions for the heat and mass transfer in flat and hollow fiber membrane-based adiabatic sorber beds.

Geometry	Parameter	
Flat membrane-based adiabatic sorber bed 	$\dot{q}\,(\xi) = \frac{k_s T_o}{\delta_s\sqrt{\pi\xi}}\bar{\theta}_{inf}\quad [\frac{w}{m^2}]$	(1)
	$\dot{m}\,(\xi) = \frac{\rho_s D_s c_o}{\delta_s}\sqrt{\frac{Le}{\pi\xi}}\,\bar{\gamma}_{inf}\quad [\frac{kg}{m^2 s}]$	(2)
Hollow fiber membrane-based adiabatic sorber bed 	$\dot{q}(\xi) = \frac{k_s T_o}{R_{HFM}}\frac{\partial(ILT(\Theta(s,\eta=1)))}{\partial\eta}\Big\vert_{inf}\quad [\frac{w}{m^2}]$	(3)
	$\dot{m}(\xi) = \frac{\rho_s D_s c_o}{R_{HFM}}\frac{\partial(ILT(\Upsilon(s,\eta=1)))}{\partial\eta}\Big\vert_{inf}\quad [\frac{kg}{m^2 s}]$	(4)
$\bar{\gamma}_{inf} = [\frac{k_m(p_v-p_o)}{k_m b_5 + 2\frac{b_3\sqrt{Le}}{\sqrt{\pi\xi}} + \frac{k_m b_4\Lambda}{\sqrt{Le}}}]$	$\bar{\theta}_{inf} = \frac{\Lambda}{\sqrt{Le}}[\frac{k_m(p_v-p_o)}{k_m b_5 + 2\frac{b_3\sqrt{Le}}{\sqrt{\pi\xi}} + \frac{k_m b_4\Lambda}{\sqrt{Le}}}]$	(5)
$\Theta(s,\eta) = \left[\frac{\Lambda}{\sqrt{Le}}\frac{I_1(\sqrt{Le.s})}{I_1(\sqrt{s})}\right]\left[\frac{\frac{k_m(p_v-p_o)}{s}}{b_3\sqrt{Le.s}I_1(\sqrt{Le.s}) + k_m b_4\frac{\Lambda}{\sqrt{Le}}\frac{I_1(\sqrt{Le.s})}{I_1(\sqrt{s})}I_o(\sqrt{s}) + k_m b_5 I_o(\sqrt{Le.s})}\right]I_o(\sqrt{s}\eta)$		(6)

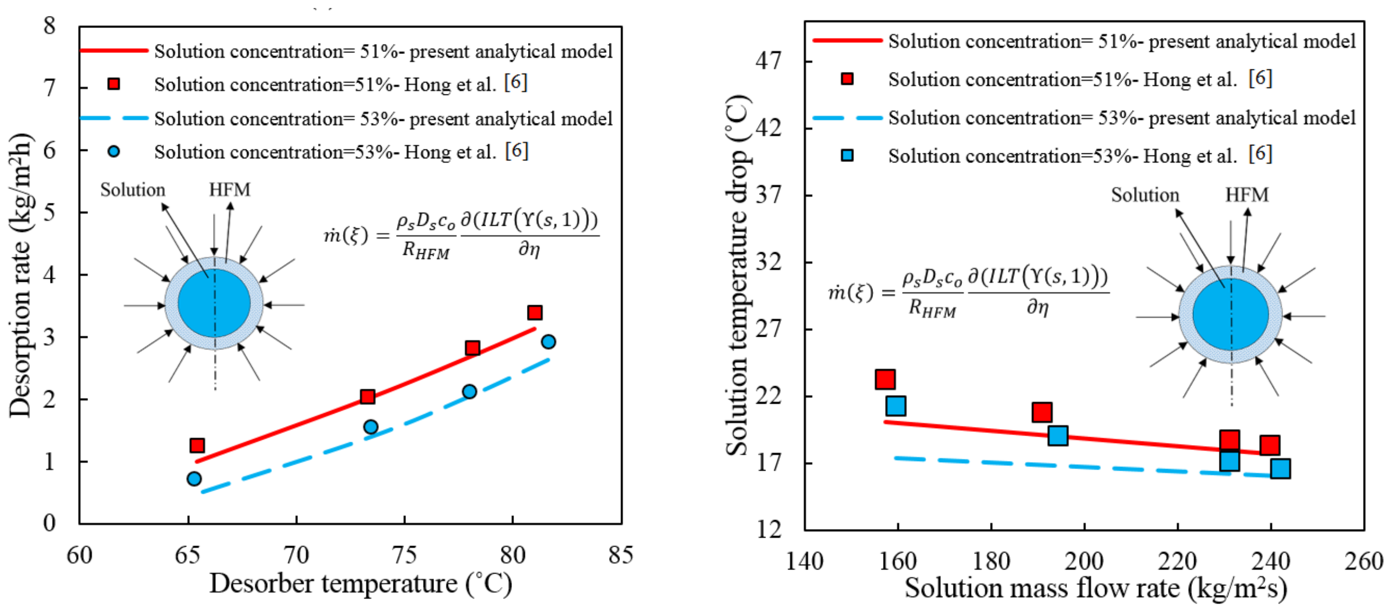
$\Upsilon(s,\eta)=\left[\dfrac{\frac{k_m(p_v-p_o)}{s}}{b_3\sqrt{Le.s}I_1\left(\sqrt{Le.s}\right)+k_m b_4\frac{\Lambda}{\sqrt{Le}}\frac{I_1\left(\sqrt{Le.s}\right)}{I_1\left(\sqrt{s}\right)}I_o\left(\sqrt{s}\right)+k_m b_5 I_o\left(\sqrt{Le.s}\right)}\right]I_o\left(\sqrt{Le.s}\eta\right)$		(7)
Inverse Laplace Transform (ILT) using the Stehfest method [3,4]		
$\theta(\xi,\eta)=\frac{ln2}{\xi}\sum_{i=1}^{N}V_i\Theta(\frac{ln2}{\xi}i,\eta)$ & $\gamma(\xi,\eta)=\frac{ln2}{\xi}\sum_{i=1}^{N}V_i\Upsilon(\frac{ln2}{\xi}i,\eta)$ & $V_i=(-1)^{\frac{N}{2}+j}\sum_{k=[\frac{j+1}{2}]}^{\min(j,\frac{N}{2})}\frac{k^{\frac{N}{2}}(2k)!}{(\frac{N}{2}-k)!\,k!\,(k-1)!\,(j-k)!\,(2k-j)!}$		(8)
Non-dimensional parameters	$\theta(\xi,\eta)=\frac{T(\xi,\eta)-T_o}{T_o},\gamma(\xi,\eta)=\frac{c(\xi,\eta)-c_o}{c_o},\xi=\frac{x}{\delta_s^2}\frac{\alpha_s}{\bar{u}}\,or\,\frac{x}{R_{HFM}^2}\frac{\alpha_s}{\bar{u}},\eta=\frac{y}{\delta_s}\,or\,\frac{r}{R_{HFM}},\Lambda=\frac{h_{abs}c_o}{c_sT_o},Le=\frac{\alpha_s}{D_s}$	

4. Model Validation

Figure 2 shows a comparison between the present models against the numerical results of Venegas et al. [5] and the experimental data of García-Hernando et al. [6] and Hong et al. [7]. The minimum, average, and maximum relative differences between the proposed model and data from Refs. [5–7] are presented in Table 2. The proposed models follow the trend and show a good agreement with the data.



(a) (b)



(c) (d)

Fig. 2. The comparison between the results from the similarity solution method used for the flat membrane-based adiabatic absorber against: (a) the numerical result of Venegas et al. [5]; and (b) the experimental data of García-Hernando et al. [6]. Also, the comparison between the present Laplace transform model for the hollow fiber membrane-based (HFM) adiabatic desorber against: (c); and (d) the experimental data of Hong et al. [7].

Table 2. The minimum, average, and maximum relative difference between the present models compared to Refs. [5–7].

Study		Min. relative difference (%)	Averaged relative difference (%)	Max. relative difference (%)
Venegas et al. [5]	Fig. 3 (a)	0	12.7	28.7
García-Hernando et al. [6]	Fig. 3 (b)	1.7	15.3	26.5
Hong et al. [7]	Fig. 3 (c)	2.2	12	29.1
	Fig. 3 (d)	5.3	7.9	12.4

Conclusion

This study proposed two analytical solutions for the coupled heat and mass transfer in flat and hollow fiber membrane-based adiabatic sorber beds for absorption heat pump/chiller application. The similarity solution and Laplace transform method were used to develop the analytical models. The presented analytical models were validated with experimental data and numerical results available in the literature with an average relative difference of 12%.

Acknowledgments
This research is supported by funding from the Pacific Institute for Climate Solutions (PICS) Opportunity Grant (No. 36170-50280) and the Natural Sciences and Engineering Research Council of Canada (NSERC) Advancing Climate Change Science in Canada Grant (No. 536076-18).

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